

Organizational Learning: A Discrete Choice Approach

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Abstract. The idea of organizational learning may be identified with the process through which the firm makes, with an increasing degree of certainty, choices about strategies to follow. A discrete choice setup is considered and applied to a dynamic process of knowledge accumulation within the firm. The most important results relate to: (i) the determination of a steady state point in which probabilities of choice take constant values; (ii) the fact that the steady state is not necessarily a stable one. If learning is subject to decreasing returns, stability holds; when knowledge accumulation exhibits increasing returns there is a divergence to a certainty strategy selection result, that is attained, at least asymptotically, in the long run.

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1. Introduction

The firm is a knowledge-creating entity. This view is shared by many authors, who see the firm as a group of people capable of generating mechanisms for mutual understanding, which are essential for increased

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efficiency and productivity. The organizational learning process should be understood as a dynamic characteristic of every organization – as firms develop their activity they learn from past mistakes and they gain the capacity to progressively make more accurate decisions.

The notion of entity engaged in knowledge creation is supported by the work of Nonaka and Takeuchi (1995), Teece, Pisano and Shuen (1997), Nahapiet and Ghoshal (1998) and Nonaka, Toyama and Nagata (2000). The main argument of these authors relates to the definition of rationality within firms; rejecting the mainstream view of neoclassical economics of full rationality, they accept the notion of bounded rationality. According to this notion, the firm takes the form of an evolving system, where wrong decisions can be taken, but where mistakes become less frequent as learning takes place. Bounded rationality replaces a passive view of the efficient firm that optimizes the use of available resources by a dynamic framework of knowledge generation. The firm creates and solves problems, generates and applies knowledge in dealing with those problems, and this tends to raise its capabilities through time.

The concept of bounded rationality to be adopted throughout the paper is essentially linked to the referred idea of learning as a means to increase the degree of rationality. However, one should keep in mind that such concept can be understood as a much deeper notion, with several important implications regarding the behaviour of economic agents. As the plentiful literature on this subject points out [see the work of prominent scholars like Simon (1982) and Kahneman (2003), among many others], we should consider that:

(i) Bounded rationality involves the use of unsophisticated methods to evaluate economic events, given the uncertain nature of the environment. For instance, the firms' decisions are boundedly rational in the sense that in the presence of uncertainties, the firm will not be able to take the full probabilistic view of the surrounding threats and opportunities;

(ii) Learning and knowledge are not the only determinants of the degree of rationality used in decision making. A firm can choose not to maximize profits even though it hypothetically has full knowledge about the market and the competitors. The policy of the firm (regarding, for instance, prices or production technologies) may be oriented just to securing a certain market share.

The idea that knowledge is created through a direct dynamic learning process is not new in economics; it goes back to Arrow (1962), Rosen (1972), Nelson and Winter (1982) and Crémer (1986). In the words of Arrow (1962),

“Knowledge has to be acquired (...) The acquisition of knowledge is what is usually termed ‘learning’ (...) Learning is the product of experience. Learning can only take place through the attempt to solve a problem and therefore only takes place during activity.” (page 155).

More recently, the debate about knowledge within organizations has focused on the development of technical languages or codes which facilitate the interaction among the firm’s constituent parts. For instance, Chowdry and Garmaise (2003) develop a theory of intrafirm communication where the communication of ideas that imply higher levels of productivity depends on the quality of the firm’s internal language. Richer languages are simultaneously a cause and a consequence of learning and knowledge accumulation – through easier communication learning processes may take place and, on the other hand, learning stimulates good quality communication. On a similar line of reasoning, Crémer(1993), Garicano (2000), Wernerfelt (2003) and Crémer, Garicano and Prat (2005) emphasize the importance of technical languages and shared meanings as a fundamental tool for increased efficiency in decision making, under a dynamic environment.

The theme of organizational learning is not dissociable from the issue of organizational memory. We have assumed in the previous paragraphs that, as stated by Argyris and Schon (1978), the organization is an independent learning organism. However, the firm is not a biological organism, it does not learn and forget as any human being. New knowledge has to be stored in order to avoid the depreciation of organizational capital [Bannon and Kari (1996), Cohen and Sproull (1996), Benkard (1999)].

According to Walsh and Ungson (1991), the knowledge that the firm possesses corresponds to stored information that can be used to take decisions on the present moment. These authors also discuss the relevance of understanding where such knowledge is stored and, thus, how likely it is for the organization to be able to keep a long memory. Organizational memory can be retained in individuals (this is the central retention location of knowledge in organizations), organizational culture and climate [see Schein (1992)], organizational change [Schein (1999)], organizational structures

[organizational design is important for the ability to retain knowledge; see Maskin, Qian and Xu (1997), Harris and Raviv (2000) and Xu (2003)], and paper or digital archives.

In this sense, the firm should not be concerned solely with learning (the creation of knowledge) but also with the retention of knowledge. Existent knowledge serves the purpose of helping to solve present problems that have some degree of resemblance with past events; redundancy can be eliminated and efficiency gains are likely to succeed. It is important for the firm to have an ability to retrieve or to recover information about past experiences to make the decision-making process easier as new situations arise.

Is a strong organizational memory beneficial for the firm in every circumstance, or can it imply some kind of drawback? Nootboom and Bogenrieder (2003) emphasize the link between knowledge and routines. As knowledge is stored, there is the risk that activities become habitual, automatic and routinized; this has the advantage of speeding the decision making process; however, it can be a problem under novel circumstances. This takes us to the issue of the depth of learning. The referred authors distinguish between exploitation and exploration of knowledge. An organization may focus on the creation of new knowledge or on the application of already existent knowledge. Given its scarce resources, a trade-off arises: too much exploration implies the absence of the use of knowledge in the generation of a return; high levels of exploitation do not allow for the creation of new knowledge, and thus obsolescence or memory loss will prevail.

The various ways that we have seen to command the discussion about organizational learning converge in one central idea: the firm is a dynamic entity that learns through time. The stock of knowledge it accumulates constitutes an important asset that is most of the time tacit, intangible, disseminated across the organization and with a residual value outside the context it is developed.

This paper tries to bring together the several notions about organizational learning and organizational forgetting that were pointed out throughout this introduction. A dynamic model is presented. The model relies on discrete choice theory, initially proposed by McFadden (1973), Manski and McFadden (1981) and Anderson, de Palma and Thisse (1993), and applied to important fields of economic and social interest by authors

like Brock and Hommes (1997, 1998) and Brock and Durlauf (2001, 2003). The setup considers a firm that has to choose between a given number of strategies in order to attain the best expected outcome. Its choice is constrained by a bounded rationality mechanism; as the firm learns with past experiences, however, the degree of rationality rises, in the sense that the learning process leads to a higher degree of certainty concerning the choice of a strategy. The problem also considers a memory loss term that reflects the organizational forgetting component of the knowledge accumulation process. The model's results will differ for different shapes of a learning function; in particular, marginal returns to learning have a central role.

The remainder of the paper is organized as follows. Section 2 presents the model's main features; section 3 discusses dynamic results; section 4 illustrates the model's dynamics through a numerical example; and section 5 synthesizes the most relevant ideas.

2. The Discrete Choice Setup

Consider a firm that in each time moment chooses a strategy for the development of its activity. There is a set of N options regarding strategy selection and each one produces a given expected outcome, $a_i(t)$, $i=1, \dots, N$. The selected strategy is not known with certainty, but it is possible to assign probabilities to the several available options. Variable $x_i(t)$ defines the probability of choice corresponding to strategy i . The choice distribution is given by vector $x(t)=[x_1(t), \dots, x_N(t)]$, with $\sum_{n=1}^N x_n(t) = 1$. Changes in the choice distribution are the result of the evolution of expected outcomes (which we assume as an exogenous component of our setup) and of a mechanism of bounded rationality, which is adapted in order to consider a learning process.

Let $Q_i(t)$ represent the interpretation the firm makes about the expected outcome of option i . This might be interpreted as a memory variable, because it translates what the firm retains regarding expected outcomes. The memory variable evolves according to the following rule,

$$\dot{Q}_i(t) = a_i(t) - \alpha Q_i(t), \quad i = 1, \dots, N; Q_i(0) = 0; \alpha \in (0,1) \quad (1)$$

In equation (1), in moment t the firm adds to its knowledge about strategy i the expected outcome of following such strategy and subtracts from that knowledge a term of memory loss. The memory loss parameter, α , represents a rate at which there is an obsolescence of accumulated knowledge concerning strategy i . In each moment of time, $Q_i(t)$ rises as a result of positive expected outcomes and falls in the extent in which the value of past rewards is forgotten. Parameter α can be designated as an entropy component of the model, in the sense that it introduces noise on the firm's choice. A memory vector includes all the memory variables: $Q(t)=[Q_1(t), \dots, Q_N(t)]$.

We must emphasize that $Q(t)$ is a vector of expected memory variables. This is just the deterministic part of the accumulated outcome measures, that is, we should define these measures, more completely, as $\tilde{Q}_i(t) = Q_i(t) + \varepsilon_i(t)$, where $\varepsilon_i(t)$ represents an IID disturbance term across $i=1, \dots, N$. The presence of noise is essential to the analysis that is undertaken below. It is by considering that $\varepsilon_i(t)$ is drawn from a double exponential distribution that we will be able to state that probabilities of choice evolve under a discrete choice mechanism. The consideration of the referred distribution for the noise term implies that in the limit, as the number of possibilities of choice goes to infinity, the probability that the firm selects strategy i will be given by the 'Gibbs' probabilities, i.e., by discrete choice function f_i , as defined below.

Probabilities of choice are updated through time. The underlying mechanism for such updating will be a discrete choice model, which, as stated in the previous paragraph, is admissible when accumulated outcomes are taken as stochastic variables, and the corresponding random component is IID and obtained from a double exponential distribution. We define function f_i such that $x_i(t) = f_i[\beta(t), Q(t)]$, with $\beta(t) \geq 0$. This function has an explicit functional form, $f_i[\beta(t), Q(t)] = e^{\beta(t) \cdot Q_i(t)} / \sum_{n=1}^N e^{\beta(t) \cdot Q_n(t)}$; nevertheless, we will only need to know, for subsequent analysis, that the function obeys the properties in definition 1.

Definition 1 – When the probability of choice is defined under a discrete choice mechanism, this probability is the output of a function f_i that obeys the following set of properties,

$$\begin{aligned}
\text{i)} & f_i[0, \mathbf{Q}(t)] = 1/N; \\
\text{ii)} & \lim_{\beta(t) \rightarrow \infty} f_i[\beta(t), \mathbf{Q}(t)] = \begin{cases} 1 & \text{if } Q_i(t) > \sum_{n=1}^N Q_n(t) \cdot x_n(t) \\ 0 & \text{if } Q_i(t) < \sum_{n=1}^N Q_n(t) \cdot x_n(t) \end{cases}; \\
\text{iii)} & \frac{\partial f_i(t)}{\partial \beta(t)} = x_i(t) \cdot \left[Q_i(t) - \sum_{n=1}^N Q_n(t) \cdot x_n(t) \right]; \\
\text{iv)} & \frac{\partial f_i(t)}{\partial Q_i(t)} = \beta(t) \cdot x_i(t) \cdot [1 - x_i(t)]; \\
\text{v)} & \frac{\partial f_i(t)}{\partial Q_j(t)} = -\beta(t) \cdot x_i(t) \cdot x_j(t), \quad j \neq i; \\
\text{vi)} & \frac{\partial f_i(t)}{\partial \beta(t)} - \frac{\partial f_j(t)}{\partial \beta(t)} = \ln x_i(t) - \ln x_j(t).
\end{aligned}$$

Variable $\beta(t)$ is known as the intensity of choice. It reflects a random effect over choices. When $\beta(t)=0$ (property i), choices are completely random: probabilities are equal across different options. As $\beta(t)$ increases, the degree of randomness is lowered: strategies with higher expected outcomes will be chosen with a higher probability. In the limit case (property ii), $x_i(t)$ is asymptotically 1 for a memory variable regarding strategy i higher than the average of all memory values and it is zero otherwise. The intensity of choice can be interpreted as translating knowledge; as the firm accumulates knowledge, it will have higher capabilities concerning strategy selection and thus a learning process is synonymous with an increasing intensity of choice. Note that there is a straightforward relation between the intensity of choice and the noise variable; in concrete, the intensity of choice is inversely related to the variance of the noise terms.

Properties (iii) to (v) are the first order derivatives of the probability function in order to the several variables. According to (iii), a higher intensity of choice implies a higher probability of choosing option i when the memory variable displays a value that is superior to the average memory values. From (iv) one observes that when the memory value concerning option i increases this has, naturally, a positive impact over the probability of choice associated to such an option. An increase in the memory value of any other strategy (v) evidently causes a contraction on the value of $x_i(t)$.

Finally, property (vi) is important for the subsequent derivation of a motion equation.

We are interested in finding a dynamic equation that represents the motion of the probability variable. Deriving function f_i in order to time, the following condition holds,

$$\dot{x}_i(t) = \frac{\partial f_i(t)}{\partial \beta(t)} \cdot \dot{\beta}(t) + \frac{\partial f_i(t)}{\partial Q_i(t)} \cdot \dot{Q}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^N \frac{\partial f_i(t)}{\partial Q_j(t)} \cdot \dot{Q}_j(t)$$

Replacing in the previous condition the partial derivatives included in definition 1 and taking in consideration equation (1), one arrives at

$$\frac{\dot{x}_i(t)}{x_i(t)} = \beta(t) \left[a_i(t) - \sum_{n=1}^N a_n(t) x_n(t) \right] - \left[\alpha - \frac{\dot{\beta}(t)}{\beta(t)} \right] \beta(t) \cdot \frac{\partial x_i(t)}{\partial \beta(t)}, \quad i=1,2,\dots,N \quad (2)$$

To study the dynamics of the model, we define variable $\phi(t) \equiv x_i(t)/x_j(t)$, with i and j two possible strategies. Accordingly, given (2) for i and j , and property (vi) in definition 1, we find

$$\frac{\dot{\phi}(t)}{\phi(t)} = \beta(t) \cdot [a_i(t) - a_j(t)] - \left[\alpha - \frac{\dot{\beta}(t)}{\beta(t)} \right] \cdot \ln \phi(t) \quad (3)$$

Equation (3) includes two components. The first is an adaptation term; in this equation, the intensity of choice arises as a velocity of adaptation variable [a high $\beta(t)$ implies a fast increase (or decrease) in the probability of choosing option i relatively to the probability of choosing option j]. The second term reflects entropy. Parameter α is the velocity of memory loss. Memory loss is attenuated by a process of learning, which is translated in the growth rate of the intensity of choice. This growth rate will be modelled in the following way,

$$\frac{\dot{\beta}(t)}{\beta(t)} = g[\beta(t)] - \delta, \quad \beta(0) = \beta_0 \text{ given.} \quad (4)$$

where $g[\beta(t)]$ is a learning function. We assume that the capability to make choices grows with the accumulated value of β . Parameter $\delta > 0$ is a

knowledge obsolescence rate; the higher the value of this rate, the more the capacity to make choices is lost through time.

The dynamics of the model are associated to the intertemporal behavior of (3) and (4). We have two endogenous variables: a ratio between probabilities of choice and an intensity of choice variable, which grows through a process of learning. The dynamic properties of the model will be dependent on the shape of function g , as one will understand below.

3. Learning Dynamics

One has assumed a learning function, which translates the notion that the capacity to make choices rises as a result of the accumulated capabilities at this level. Thus, organizations follow through time a learning process. As the intensity of choice becomes a higher value, the firm gains in terms of knowledge about the certainty with which it takes decisions. The learning function may reflect the prevalence of increasing returns or, alternatively, decreasing returns. Considering a parameter $b > 0$, function g is defined in the following way.

Definition 2 – Function $g[\beta(t)]$ reflects a learning process. In particular, g corresponds to average accumulated knowledge. Function $g: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is continuous, monotonic and (n -order) differentiable. For this function we assume a constant ratio between marginal and average learning, such that $b = \frac{g'}{g} \cdot \beta(t) + 1$, with g' the first-order derivative of the average knowledge function. If $b > 1$, learning exhibits increasing returns; for $b < 1$, decreasing returns to learning are observable.

Under definition 2, we present the steady state result.

Proposition 1 – The proposed choice-learning system entails a unique steady state point, for $b \neq 1$.

Proof: The steady state is a point $(\bar{\beta}, \bar{\phi})$ obtained for the absence of growth of the two endogenous variables. Solving the system (3)-(4) under

the constraint $\frac{\dot{\phi}(t)}{\phi(t)} = \frac{\dot{\beta}(t)}{\beta(t)} = 0$, the following long term results are achieved, $g(\bar{\beta}) = \delta$ and $\bar{\phi} = e^{(\bar{\beta}/\alpha).(\bar{a}_i - \bar{a}_j)}$.

Because g is a continuous and monotonic function, the only constraint that is necessary to impose for the existence of the steady state point is indeed $b \neq 1$. Once this constraint is verified (if there are no constant returns to learning), one guarantees a unique $\bar{\beta}$ value; function $g(\bar{\beta})$ intercepts the horizontal line δ in only one point, when assumed the referential $[\bar{\beta}, g(\bar{\beta})]$. Assuming that unique steady state values exist for the expected reward variables, it is straightforward to realize that the equilibrium probabilities of choice ratio are also unique values.

Note the influences over the steady state value of variable $\phi(t)$. The probability of choosing strategy i instead of choosing strategy j rises when the equilibrium value of the expected reward regarding option i grows relative to the expected reward of option j in the long run equilibrium position. A high intensity of choice is synonymous with certainty ($\bar{\phi}$ becomes close to zero for $\bar{a}_i < \bar{a}_j$, and it becomes a significantly higher than 1 value for $\bar{a}_i > \bar{a}_j$). A significant value of memory loss implies a steady state probabilities ratio near 1; that is, it is hard to distinguish between strategies in the correspondent selection process.

The stability of the steady state depends on the sign of parameter b .

Proposition 2 – Increasing returns to learning ($b > 1$) imply saddle-path stability. Global stability is guaranteed under decreasing returns to learning ($b < 1$).

Proof: To prove proposition 2, we linearize system (3)-(4) in the steady state neighbourhood,

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{\phi}(t) \end{bmatrix} = J \cdot \begin{bmatrix} \beta(t) - \bar{\beta} \\ \phi(t) - \bar{\phi} \end{bmatrix}, \quad J = \begin{bmatrix} (b-1).\delta & 0 \\ \frac{(b-1).\delta + \alpha}{\alpha} . (\bar{a}_i - \bar{a}_j) . \bar{\phi} & -\alpha \end{bmatrix}$$

The Jacobian matrix, J , has as elements the first-order derivatives of each differential equation in order to each endogenous variable, being these derivatives evaluated in the steady state vicinity. Stability results are derived through the calculus of eigenvalues; in the present case, it is straightforward that these are $\lambda_1 = (b-1)\delta$ and $\lambda_2 = -\alpha$. While the second eigenvalue is always a negative quantity, the sign of the first eigenvalue is either positive or negative depending on the value of b . If $b > 1$, then $\lambda_1 > 0$; if $b < 1$, then $\lambda_1 < 0$. Since negative eigenvalues correspond to stability dimensions of the system and positive eigenvalues respect to instability dimensions, under increasing returns to learning there is a unique stability dimension, reflecting saddle-path stability, while under decreasing returns the system possesses a dimension-two stable area, and thus global stability holds.

The computation of eigenvectors associated to eigenvalues λ_1 and λ_2 allows for the determination of two trajectories, through which variables converge or diverge relatively to the steady state position. These trajectories take the following form:

$$\phi(t) = \bar{\phi} \cdot (1 - \ln \bar{\phi}) + \frac{\bar{a}_i - \bar{a}_j}{\alpha} \cdot \bar{\phi} \cdot \beta(t) \quad (\lambda_1);$$

$$\beta(t) = \bar{\beta} \quad (\lambda_2).$$

For $b < 1$ both trajectories represent convergence paths. Given an initial point $[\beta(0), \phi(0)]$, both variables will converge to the steady state through a movement of approximation to one of the two trajectories.

When increasing returns to learning prevail, only the second of these trajectories represents a stable arm, being the first trajectory an unstable path. Saddle-path stability implies that the stable arm is followed only if $\beta(0)$ is in the vicinity of $\beta(t) = \bar{\beta}$; for $\beta(t) \neq \bar{\beta}$, instability will prevail, that is, the two variables will follow a path that leads them, asymptotically, to the unstable trajectory. The unstable result implies that alternatively to an intermediate result under which strategies i and j both will exhibit a positive and less than one probability of choice, we will have a steady state result in which one of the options is followed with probability one.

Note that if the expected outcome of option i is higher than the expected outcome of option j , the unstable trajectory will be positively sloped, and the learning process implies an everlasting growth of variable

$\phi(t)$ in the direction of $x_i(t)=1$ and $x_j(t)=0$. For $\bar{a}_i < \bar{a}_j$, the unstable path is negatively sloped and the increase in choice capabilities is synonymous of a fall in the value of $\phi(t)$, meaning the convergence to the point where $x_i(t)=0$ and $x_j(t)=1$ (in the absence of a third, alternative, strategy).

4. Numerical Example

To illustrate the model's results, we present in this section a small numerical exercise. We consider a specific functional form for the learning function, which obeys the requirements imposed by definition 2; this function is $g[\beta(t)] = G / \beta(t)^{1-b}$, with $G > 0$. We also assume that rewards are constant over time for each of the available strategies; consider, in particular, that $a_i = 0.5$ and $a_j = 0.48$. Relative to the other parameters, one takes, for now, $\alpha = 0.1$ and $\delta = 0.05$; and, concerning the production function, $G = 0.1$ and $b = 0.75$. Given the value chosen for b , we begin by analyzing the decreasing returns – stable node outcome.

In a first stage, one determines steady state values for the intensity of choice and for the ratio of probabilities. Note that $G / \bar{\beta}^{1-b} = \delta$ is a steady state relation and, thus, $\bar{\beta} = (G / \delta)^{1/(1-b)} = 16$. The long run ratio between probabilities of choice is presented in the proof of proposition 1: $\bar{\phi} = e^{(\bar{\beta}/\alpha) \cdot (a_i - a_j)} = 24.53$. This value is larger than 1 because the expected reward from choosing strategy 1 is higher than the reward relating to the second strategy, which implies that the probability of choice associated to the first option has to be higher (in the case, if only two strategies are available then, for $\bar{\phi} = 24.53$ and $\bar{x}_i + \bar{x}_j = 1$, we will have $\bar{x}_i = 96\%$ and $\bar{x}_j = 4\%$).

In the described case, independent from the initial values $\beta(0)$ and $\phi(0)$, the system converges to the steady state $(\bar{\beta}, \bar{\phi}) = (16; 24.53)$. In terms of local dynamics, one can observe that the Jacobian matrix is $J = \begin{bmatrix} -0.0125 & 0 \\ 0.4293 & -0.1 \end{bmatrix}$. The two negative eigenvalues that we have acknowledged in the generic case are observed in the matrix ($\lambda_1 = -0.0125$ and $\lambda_2 = -0.1$), and two stable trajectories are computable in our two dimensional space, making the system stable independent from the initial state. The referred trajectories, presented in their general form in section 3,

are $\phi(t) = -53.963 + 4.906 \cdot \beta(t)$ and $\beta(t)=16$. We sketch local dynamics in figure 1.

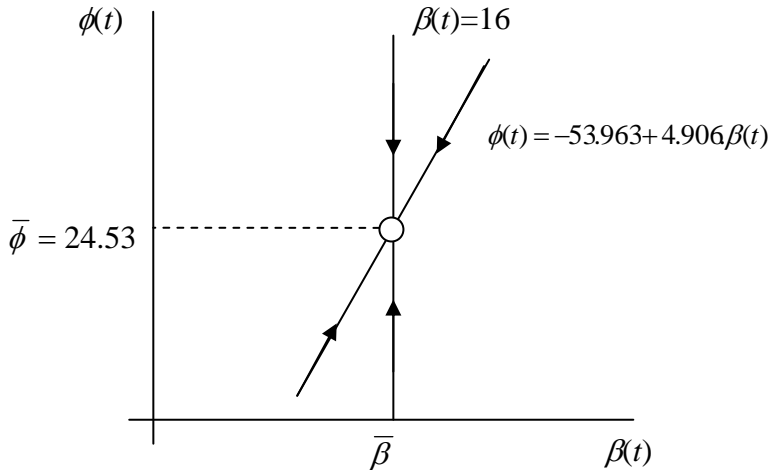


Figure 1 – Local dynamics, for the case $b < 1$.

In our simple model, local dynamics coincide with global dynamic behaviour, as can be confirmed by looking at figures 2 to 4.1 These figures represent, for the specific presented parameter values, time trajectories for both endogenous variables, and also a diagram relating to the joint convergence behaviour of the variables towards the steady state. The figures are drawn for initial values $\beta(0)=12$ and $\phi(0)=25$. With these figures, one can visualize the transition process towards the steady state for a specific set of parameter values.

¹ These figures are drawn using iDMC (interactive Dynamical Model Calculator). This is a free software program available at www.dss.uniud.it/nonlinear, and copyright of Marji Lines and Alfredo Medio. The same software is used to plot figures 5 and 7.

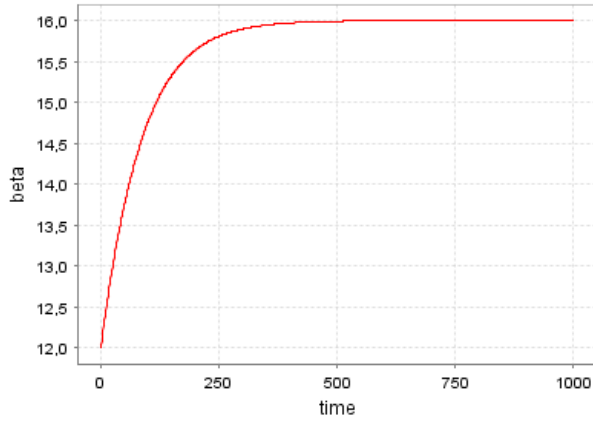


Figure 2 – Transitional dynamics, for the case $b < 1$: variable β .

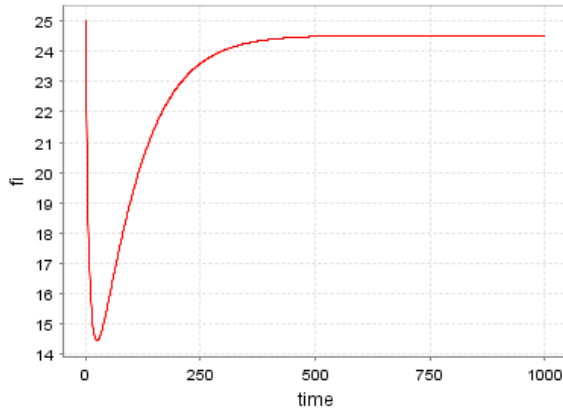


Figure 3 – Transitional dynamics, for the case $b < 1$: variable ϕ .

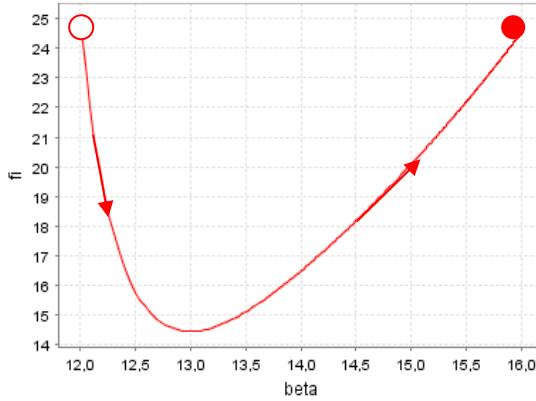


Figure 4 – Transitional dynamics, for the case $b < 1$: relation between both variables.

Changes in parameter values will modify the steady state and the way the convergence to the long run position is fulfilled, but no change will occur in the qualitative properties of the system unless parameter b rises above 1. As an example, consider that the degree of entropy rises from $\alpha=0.1$ to $\alpha=0.25$. If everything else remains the same, this parameter change does not modify significantly the previous outcomes, as figure 5 illustrates.

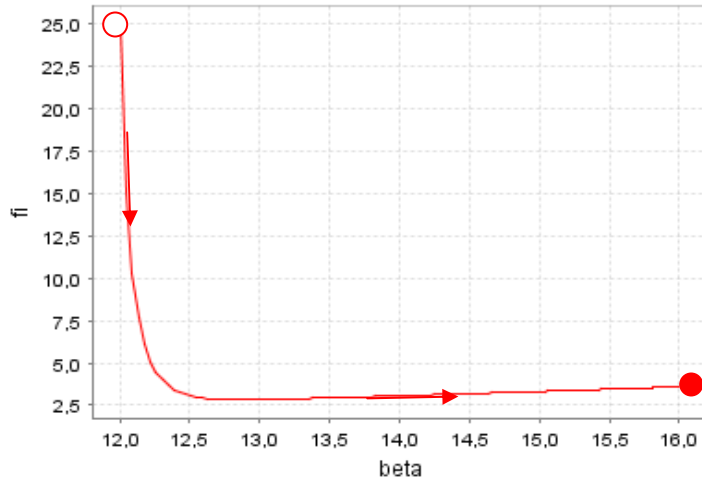


Figure 5 – Transitional dynamics, for the case $b < 1$ ($\alpha=0.25$): relation between both variables.

When $b > 1$, the previous global analysis is not possible, unless one considers that the system is, in $t=0$, over the stable arm, which is now the one dimensional condition $\beta(t) = \bar{\beta}$. Continuing to consider the various parameter values initially proposed, except b that will be in this case $b=1.25$, we observe that the steady state is $\bar{\beta} = 0.0625$ and $\bar{\phi} = 1.0126$. The presence of increasing returns implies a much smaller intensity of choice value and, as a consequence, the ratio between probabilities becomes much closer to one (nevertheless, because we have not changed the assumption $a_i > a_j$, it remains a value higher than one). In this new scenario, $\bar{x}_i = 50.3\%$ and $\bar{x}_j = 49.7\%$.

The obtained steady state probabilities allow the inference that increasing returns to learning produce a steady state with a much lower level

of accumulated knowledge (the intensity of choice is near zero), what seems to be counter intuitive. What one should note is that this steady state will hardly be attained; this would require $\beta(0)$ exactly equal to 0.0625. The saddle-path equilibrium indicates that the probable outcome will be one in which $\phi(t)$ tends to zero or to infinity, that is, $x_i(t)$ tends to zero or to one (and vice-versa for the other probability). The Jacobian matrix is now

$$J = \begin{bmatrix} 0.0125 & 0 \\ 0.0228 & -0.1 \end{bmatrix}$$

and the unstable trajectory corresponds to the expression $\phi(t) = 1 + 0.2025 \cdot \beta(t)$.

A phase diagram similar to the one in figure 1 is presented in figure 6.

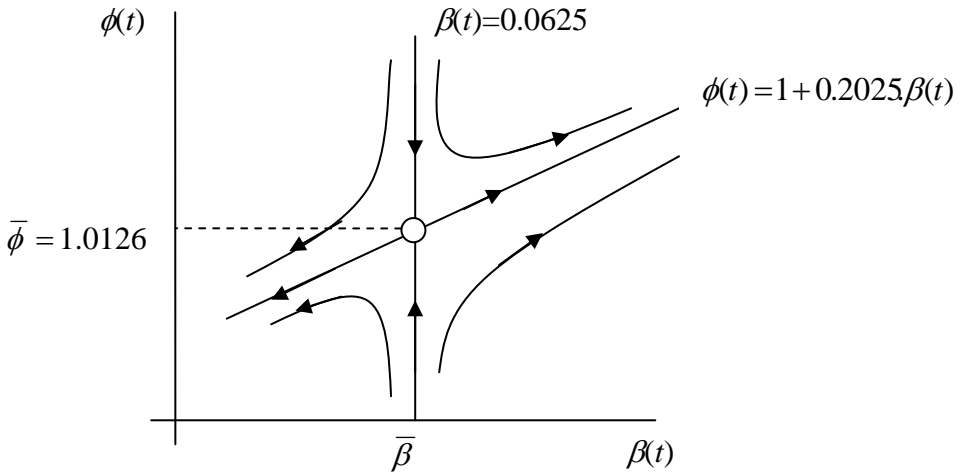


Figure 6 – Local dynamics, for the case $b > 1$.

Figure 6 gives the indication that if $\beta(0) < \bar{\beta}$, then $\bar{\phi} \rightarrow 0$ and that if $\beta(0) > \bar{\beta}$ (which is the most likely case, since $\bar{\beta}$ is very small), then $\bar{\phi} \rightarrow +\infty$, i.e., $\bar{x}_i \rightarrow 1$. Figure 7 considers $\beta(0)=0.1$ and $\phi(0)=1$ to draw a specific divergence process in the space of variables.

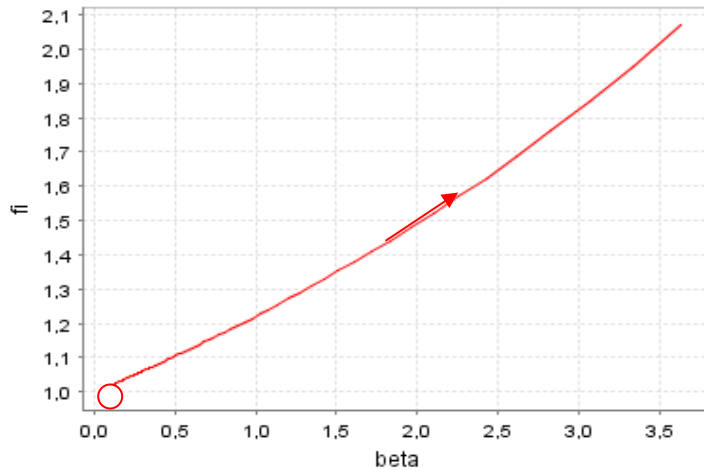


Figure 7 – Transitional dynamics, for the case $b > 1$: relation between both variables.

Any change in parameter values for the case of increasing returns to learning will not modify the qualitative nature of the associated dynamics. The equilibrium point will move, but it will hardly represent the state of the system in the long run.

5. Final Remarks

Despite the observable differences, firms can be compared to living organisms. As they develop their activities they acquire learning skills, that is, the more choices they make the better equipped they will become to make future choices. This argument allows for the consideration of a learning function and for the assumption of a knowledge accumulation process. The accumulation of knowledge is translated into an intensity of choice variable, which in turn is supposed to mean the capability of the firm in making with a high degree of precision a selection of strategy that reflects the weighting of expected rewards. The learning process is a way of reducing probabilities regarding the choice of strategies that are not evaluated as the best performing strategies. The accumulation of knowledge serves the purpose of attenuating the effects of entropy or memory loss, making it easier for the organization to choose the strategy with the higher expected return.

The undertaken analysis has allowed for the determination of a steady state point. This point, if stable, reflects a long run scenario where no growth of knowledge exists. Decreasing returns to learning imply this long term

result, which also means that positive (smaller than one) probabilities for the choice of several alternatives will be found in the steady state. On the contrary, if one assumes increasing returns to learning, unless exceptional circumstances are met, the long run result will be one of total certainty on strategy selection; as the intensity of choice increases through time, the choice becomes progressively less ambiguous and, asymptotically, the entropy effect will disappear.

The fundamental question we may ask is: which is the model's version that best fits empirical evidence? Both are admissible depending on the firm's activity and on the economic environment. We may support the stability result on the evidence that the space for generating knowledge is scarce. Learning over past knowledge reduces the available set of knowledge where learning is possible. On the other hand, it is true that knowledge generates spillovers. As learning takes place the knowledge space grows and an increasing learning ability can be evidenced. Newton's 'shoulders of giants' effect is a possible explanation for the prevalence of increasing returns to learning inside organizations. This is an important mechanism concerning efficiency, because as firms accumulate knowledge about how to make important choices regarding the strategies to follow, these choices are made with a higher degree of certainty.

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