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Constitutional Rights and Pareto Efficiency

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Abstract. This paper presents a sufficient condition under which constitutional rights lead to socially rational and Pareto efficient outcomes. The condition states that if the gains every constitutional right creates are either large enough to be positive in aggregate for the society or large enough to be maximal for some individuals, there is no conflict between constitutional rights and Pareto efficiency that generates irrational social outcomes.

Key Words: Constitutional rights; Pareto efficiency.

I. Introduction

The need for constitutional rights, as described in Mueller (1996), stems in part from "the likely preference intensity differences between individuals who undertake an action and those adversely affected by it." For instance, in cases where some sections of the society disapprove, however mildly, the religious, life style-specific or uniquely personal practices of certain individuals who feel strongly about the freedom to engage in such practices, a constitutional right may need to be invoked to protect those individuals' freedoms against the tyranny or strategic manipulation by others. The exercise of such rights, however, which may differentially affect individuals in the society through the gains or losses it produces, could lead to social irrationality inducing conflicts with Pareto efficiency.[2] Such social irrationality or Pareto inefficiency generating possibilities pose the question of under what conditions constitutional rights lead to Pareto efficient and socially rational outcomes. This paper seeks to provide an answer to this question.

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[2] By 'social irrationality,' we mean cyclical social preference over at least one subset of the domain of alternatives.

II. The General Framework

Let E be the set of a finite number of individuals forming a society, and let Z be the set of mutually exclusive social alternatives. Assume that the cardinalities of E and Z, denoted by, respectively, $|E|$ and $|Z|$, are finite, and $|E|>1$, $|Z|>2$. Each individual i in the society has a preference ordering R^i, which is a binary relation on Z such that $R^i \subseteq \{(x,y): x, y$ are in $Z\}$, and i = 1,...,n. For any x, y in Z, $(x,y) \in R^i$ means the same thing as xR^iy which will be interpreted as "x is preferred to y" by individual i. Define strict preference (P^i) and indifference (I^i) relations on $\{x,y\}$ as follows: xP^iy if and only if xR^iy and not yR^ix. xI^iy if and only if xR^iy and yR^ix.

A preference relation R^k on Z is said to be *complete* if and only if xP^ky or yP^kx or xI^ky for all x, y in Z such that $x \neq y$. R^k on Z is *incomplete* if it is not complete. R^k on Z *is transitive* if and only if for all x, y, z in Z, (xP^ky and yP^kz implies xP^kz), and (xI^ky and yI^kz implies xI^kz).[3] R^k on Z is *intransitive* if it is not transitive. R^k on Z is *acyclical* over an *m-set*[4] $\{x_1,...,x_m\}$ in Z if and only if the following condition holds: for all $x_1,....,x_m$ in Z, if $[x_1P^kx_2$, and $x_2P^kx_3$, and ... and $x_{m-1}P^kx_m]$, then $x_1R^kx_m$. R^k is *cyclical* over $\{x_1,...,x_m\}$ in Z if $x_1P^kx_2$, and $x_2P^kx_3$, and ... and $x_{m-1}P^kx_m$ and $x_mP^kx_1$. Clearly, if R^k is not cyclical over ***any m-set*** in Z, none of its subsets is.

$x \in Z$ is said to *dominate* $y \in Z$ with respect to R^k if xP^ky (i.e., if $(x,y) \in R^k$ and $(y,x) \notin R^k$). $x \in Z$ is said to be a *dominated alternative* with respect to R^k if there is a $z \in Z$ such that zP^kx. $x \in Z$ is said to be an *undominated alternative* with respect to R^k if there is no $z \in Z$ such that zP^kx. If R^k over an *m-set* is not cyclical, there is at least one undominated alternative with respect to R^k in that *m-set*.

Let h be a *collective choice rule*, that is, a mapping from the set of individual preference relations $R^{ind} = \{(R^1,...,R^n): R^k$ is a individual preference relation on Z, $k = 1,...,n\}$ into a set of preference relations $R^{soc} = \{R: R$ is a social preference relation on $Z\}$ such that for any configuration of

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[3] If R^k is complete, then the conditions specified in the definition of transitivity imply the following two properties: For all distinct x,y,z *in* Z,

xP^ky and yI^kz *imply* xP^kz,

xI^ky and yP^kz *imply* xP^kz (Sen, 1970:19).

Alternatively, R^k on Z is transitive if and only if for all x, y, z in Z, (xR^ky and yR^kz) imply xR^kz.

[4] By an m-set, we mean a set of m alternatives.

individual preference relations $R^1,...,R^n$, one and only one social preference relation R is determined, i.e. $h: R^{ind} \hookrightarrow R^{soc}$ such that $R = h(R^1,...,R^n)$. The social preference relation R is a binary relation whose strict preference and indifference parts are P and I.

Suppose that society has a constitution that defines, among other things, rights to be granted to individuals. An individual k is said to have a *constitutional right* to an alternative x_i as opposed to (against) x_j, if the constitution stipulates that x_iPx_j whenever $x_iP^kx_j$. Alternatively, we say that individual k has a constitutional right over the pair $\{x_i,x_j\}$ in Z where x_i is said to be a *right-chosen alternative*. A constitutional right over $\{x_i,x_j\}$ is said to *override* a constitutional right over $\{x_k,x_s\}$ if by the assignment of constitutional rights x_iPx_k, i.e. if someone in E, say g, with a preference $x_iP^gx_k$, has a right over $\{x_i,x_k\}$. Constitutional rights are said to be *non-overriding* if no constitutional right overrides the other.

The importance individuals attach to certain rights has a lot to do with the intensity with which they prefer right-chosen alternatives to others. Preference intensities for such alternatives are, therefore, likely to serve as a key to the understanding and resolution of the conflicts over those rights.[5] The resolution we will propose in this paper will thus be based on an explicit account of preference intensities which we will introduce as follows: let a^k_j be a utility index representing the number of real-valued utility points individual k assigns to alternative x_j in Z. Assume that individuals assign utility points to every alternative in Z. The number of utility points individual k assigns to an alternative reflects her intensity of preference for that alternative. Thus, the more intense is her preference for an alternative, the greater is the number of utility points she assigns to that alternative. For instance, if she prefers $x_i \in Z$ to $x_j \in Z$, then $a^k_i \geq a^k_j$.

Define a specific collective choice rule, which we will call *the utilitarian rule*, which takes into account the individuals' preference intensities over m alternatives in the following manner:

$$x_iPx_j \text{ iff } \sum^n_{k=1} a^k_i > \sum^n_{k=1} a^k_j$$

$$x_iIx_j \text{ iff } \sum^n_{k=1} a^k_i = \sum^n_{k=1} a^k_j,$$

where P and I indicate, respectively social strict preference and social indifference. Alternatively,

$$x_iRx_j \text{ iff } \sum^n_{k=1} a^k_i \geq \sum^n_{k=1} a^k_j,$$

[5] There are some works in the literature, such as Ng (1971) and Mueller (1996), which make use of preference intensities in their analysis of rights.

where R indicates social preference.

Finally, we define the gains and losses constitutional rights could create for individuals in *E*. A constitutional right of an individual, say *g*, over $\{x_i,x_j\}$ is said to create, for individual *g*, a gain of $|a^g_i-a^g_j|$. The exercise of the right over $\{x_i,x_j\}$ by individual *g* is said to create, for individual *k, k…g, k=1,...,n,* a gain of $|a^k_i-a^k_j|$ if $a^k_i-a^k_j$ is positive, and a loss of $|a^k_i-a^k_j|$ if $a^k_i-a^k_j$ is negative. The absolute value of the sum of all gains (losses) the right over $\{x_i,x_j\}$ creates is called the aggregate gain (loss). The net aggregate gain of the right over $\{x_i,x_j\}$ for individuals in *E* is the difference between its aggregate gain and aggregate loss. A right of *g* over $\{x_i,x_j\}$ is said to create 'maximal gain' for *g* if $a^g_i-a^g_j$ is greater than or equal to her possible gain over any other pair in *Z*. Clearly, if the right of *g* over $\{x_i,x_j\}$ creates maximal gain , x_i must be an undominated alternative with respect to R^g.

III. Constitutional Rights And Pareto Efficiency: A Resolution

To set the stage for an analysis of possible conflicts between constitutional rights and Pareto efficiency, we will pose three conditions that are set-theoretically similar to those of Sen (1970:87):

Condition *U* (Unrestricted Domain): Every logically possible combination of individual preference orderings is included in the domain of the collective choice rule.

Condition *P* (Pareto Efficiency): Let $\{x,y\}$ be any pair contained in *Z.* If for every *k* in *E* xP^ky, then xPy.

Condition *C* (Constitutional Rights): There are at least two pairs of alternatives over which constitutional rights exist. Constitutional rights are non-overriding.

It is straightforward to prove that a Sen-type result also applies to the three conditions above, i.e., given an unrestricted domain of individual preferences, there is a conflict between constitutional rights and Pareto efficiency that generates cyclical social preferences. For a formal treatment of the subject, we will first present a formal definition of the conflict in question:

Conflict between Constitutional Rights and Pareto Efficiency: For a given configuration of individual preferences, a *conflict* is said to exist between Condition *C* and Condition *P* with respect to an *m-set, m>2*, in *Z* if the simultaneous (joint) application of both conditions results in a social preference relation *R* that is cyclical over that *m-set* while the individual application of each condition in the absence of the other does not.[6]

Given an unrestricted domain of individual preferences, there exists at least one *m-set* in *Z* with respect to which a conflict exists between Condition *C* and Condition *P*. However, there are conditions under which such conflicts cease to exist. The following theorem presents such a condition that eliminates such conflicts in the entire domain of alternatives.

Theorem 3.1: If every constitutional right creates either positive net aggregate gains for the society or maximal gains for some individuals in *E*, there is no conflict between constitutional rights (Condition *C*) and Pareto efficiency (Condition *P*) with respect to any *m-set* in *Z*.

Proof: Let R^{c1} and R^{c2} denote, respectively, the sets of ordered pairs over which the social preference is determined, respectively, by constitutional rights that generate maximal gains for some individuals, and constitutional rights that generate positive net aggregate gains for the society. Let R^p denote the set of ordered pairs over which social preference relation is determined by Condition *P*. By Condition *C*, at least one of R^{c1} and R^{c2} must be non-empty. We will first consider the case where both are non-empty and deal later with the case where one of them could be empty. Regarding R^p, in cases where R^p is empty, the social preference would be determined by Condition *C* alone without any role by Condition *P*, and hence, by definition, in such cases the question of conflict between Condition *C* and Condition *P* does not arise. Hence, we need to consider only the cases where R^p is non-empty.

We will prove that, under the condition stated in the theorem, no non-empty subset of $R^{c1} \cup R^{c2} \cup R^p$ is cyclical over any *m-set* in *Z*, implying that there is no conflict between constitutional rights and Pareto efficiency with respect to any *m-set* in *Z*. We will decompose the non-empty subsets of $R^{c1} \cup R^{c2} \cup R^p$ (including itself) into three groups, and prove that no subset of $R^{c1} \cup R^{c2} \cup R^p$ in any of these groups is cyclical over any *m-set* in *Z* (the term, "subset," as it is used below, refers to non-empty subsets).

Group 1. Subsets of $R^{c1} \cup R^{c2} \cup R^p$ that are subsets of R^{c1}: by definition, constitutional rights are non-overriding, hence, no right-chosen alternative

[6] Rights over the pairs in an m-set can be assigned to individuals in many different ways. In order for a conflict to exist between Condition C and Condition P, the condition stated in the definition above should hold for at least one possible way of assigning constitutional rights.

could dominate the other. Therefore, there is at least one undominated alternative with respect to each subset of R^{c1}, implying that none of these subsets is cyclical over any *m-set* in *Z*.

Group 2. Subsets of $R^{c1} \cup R^{c2} \cup R^p$ that are subsets of $R^{c2} \cup R^p$: let R^* be the social preference relation induced by the utilitarian rule. We will first show that $R^{c2} \subset R^*$ and $R^p \subset R^*$.

Let $(x_s,x_t) \in R^{c2}$. Then, there is an individual in *E*, say *1*, who has a right over $\{x_s,x_t\}$ and who strictly prefers x_s to x_t. Such a right creates a gain of $a^1_s - a^1_t$ for individual *1*. Without loss of generality, assume that the right over $\{x_s,x_t\}$ also creates gains for individuals *2* to *h*, losses for individuals *h+1* to *m*, and neither gains nor losses for individuals *m+1* to *n*. By assumption, the net aggregate gains of the right over $\{x_s,x_t\}$ are greater than zero, implying that the aggregate gains the right over $\{x_s,x_t\}$ creates are greater than the aggregate losses it induces. Thus,

$$|\sum^h_{k=1} (a^k_s - a^k_t)| > |\sum^m_{k=h+1} (a^k_s - a^k_t)|$$

$$\Rightarrow \sum^h_{k=1} (a^k_s - a^k_s) > - \sum^m_{k=h+1} (a^k_s - a^k_t)$$

$$\Rightarrow \sum^h_{k=1} (a^k_s - a^k_t) + \sum^m_{k=h+1} (a^k_s - a^k_t) > 0.$$

Since the right over (x_s,x_t) creates neither gain nor loss for individuals *m+1* to *n*, $\sum^n_{k=m+1} (a^k_s - a^k_t) = 0$. Thus, adding this zero-sum to the left-hand side of the inequality does not affect the inequality, i.e.,

$$\sum^h_{k=1} (a^k_s - a^k_t) + \sum^m_{k=h+1} (a^k_s - a^k_t) + \sum^n_{k=m+1} (a^k_s - a^k_t) > 0$$

$$\Rightarrow \sum^n_{k=1} (a^k_s - a^k_t) > 0$$

$$\Rightarrow \sum^n_{k=1} a^k_s > \sum^n_{k=1} a^k_t,$$

which implies that $(x_s,x_t) \in R^*$. Thus, $(x_s,x_t) \in R^{c2}$ implies $(x_s,x_t) \in R^*$, i.e., $R^{c2} \subset R^*$.

Let $(x_i,x_j) \in R^p$. Then, $a^k_i > a^k_j$ for every *k* in *E*. Thus, $\sum^n_{k=1} a^k_i > \sum^n_{k=1} a^k_j$, which implies that $(x_i,x_j) \in R^*$. Hence, $(x_i,x_j) \in R^p$ *implies* $(x_i,x_j) \in R^*$, i.e., $R^p \subset R^*$. Since $R^{c2} \subset R^*$ and $R^p \subset R^*$, $(R^{c2} \cup R^p) \subset R^*$.

It is straightforward to show that R^* is transitive over every triple in *Z*: Take an arbitrary triple $\{x_1,x_2,x_3\}$ in *Z* and let $(x_1,x_2) \in R^*$ and $(x_2,x_3) \in R^*$. Then, $\sum^n_{k=1} a^k_1 \geq \sum^n_{k=1} a^k_2$ and $\sum^n_{k=1} a^k_2 \geq \sum^n_{k=1} a^k_3$. Thus, $\sum^n_{k=1} a^k_1 \geq \sum^n_{k=1} a^k_3$, which implies *that* $(x_1,x_3) \in R^*$. Therefore R^* is transitive over $\{x_1,x_2,x_3\}$. This property of R^* holds for every triple in *Z*. Since R^* is transitive over every triple in *Z*, it is acyclical over every *m-set* in *Z*. Thus, none of the subsets of R^* is cyclical over any *m-set* in *Z*. Since $R^{c2} \cup R^p$ is a subset of R^*, it is not cyclical over any *m-set* in Z, and hence neither is any of its subsets (and thus there is at least one undominated alternative with respect to each of these subsets).

Group 3. Subsets of $R^{c1} \cup R^{c2} \cup R^{p}$ that are the unions of a subset of R^{c1} and a subset of $R^{c2} \cup R^{p}$: let there be a constitutional right, of say individual t in E, over a pair $\{x_i, x_j\}$ such that $(x_i, x_j) \in R^{c1}$. By the assumption of non-overriding constitutional rights, x_i is an undominated alternative with respect to R^{c1}. By the definition of R^{c1}, the right over $\{x_i, x_j\}$ creates maximal gains for individual t, implying that x_i is an undominated alternative with respect to R^{t}. Since $R^{p} \subset (\cap^{n}_{k=1} R^{k}) \subset R^{t}$, x_i, which is an undominated alternative with respect to R^{t}, is also undominated with respect to R^{p} (and thus with respect to each of its subsets). Hence, every undominated alternative with respect to R^{c1}, such as x_i, turns out to be undominated with respect to R^{p} as well. On the other hand, again by the assumption of non-overriding constitutional rights, every right-chosen alternative is undominated. Hence, every undominated alternative with respect to R^{c1} is also undominated with respect to R^{c2} and vice versa. Therefore every undominated alternative with respect to R^{c1}, which is, by the argument above, undominated with respect to R^{c2} and R^{p}, is bound to be undominated with respect to $R^{c2} \cup R^{p}$, and hence with respect to each of its subsets.

Now, let's examine an arbitrary subset of $R^{c1} \cup R^{c2} \cup R^{p}$ that is the union of a subset of R^{c1} and a subset of $R^{c2} \cup R^{p}$. Let A be a subset of $R^{c2} \cup R^{p}$ and B be a subset of R^{c1}. By the argument relating to Group 1, no subset of R^{c1} is cyclical over any *m-set* in Z. Thus, there is at least one undominated alternative with respect to each subset of R^{c1}, and hence with respect to B. Let $x_m \in Z$ be an undominated alternative with respect to B, which is, by the argument above, also an undominated alternative with respect to any subset of $R^{c2} \cup R^{p}$, and hence with respect to A. x_m, which is an undominated alternative with respect to A and B, is bound to be an undominated alternative with respect to $A \cup B$. Since we have chosen an arbitrary subset of R^{c1} and formed its union with an arbitrary subset of $R^{c2} \cup R^{p}$, the result holds for every subset that is a union of a subset of R^{c1} and a subset of $R^{c2} \cup R^{p}$. Thus, there is at least one undominated alternative in Z with respect to each subset of $R^{c1} \cup R^{c2} \cup R^{p}$ that is a union of a subset of R^{c1} and a subset of $R^{c2} \cup R^{p}$, i.e., with respect to each subset in Group 3.

Thus, by the arguments relating to Group 1, 2, and 3 above, there is at least one undominated alternative in Z with respect to each subset in all three groups, i.e. with respect to every non-empty subset of $R^{c1} \cup R^{c2} \cup R^{p}$, which implies that neither $R^{c1} \cup R^{c2} \cup R^{p}$ nor any of its subsets is cyclical over any *m-*

set in Z.

The proof above covers the cases where both R^{c1} and R^{c2} are non-empty. To extend the proof to the cases where one of R^{c1} and R^{c2} is empty, let G^1 and G^2 be the sets of all subsets of $R^{c1} \cup R^{c2} \cup R^p$ where respectively R^{c1} and R^{c2} is empty and let G^* be the set of all subsets of $R^{c1} \cup R^{c2} \cup R^p$ where both R^{c1} and R^{c2} are non-empty. Since $G^1 \subset G^*$ and $G^2 \subset G^*$ and since, as proved above, G^* does not contain any subset that is cyclical over any *m-set* in Z, neither do G^1 and G^2.

Q.E.D.

IV. Concluding Remarks

The theorem is relevant to and significant for the constitutional discourse for two reasons. First, it establishes that whether an assignment of constitutional rights leads to a social irrationality generating conflict with Pareto efficiency has a lot to do with the gains and losses those rights create for the society as a whole or for particular individuals. If the gains of any constitutional right are large enough to be positive in aggregate, or large enough to be maximal for some individuals, the resulting social choices are bound to both socially rational and Pareto efficient. Second, it provides insights into the nature of externalities that give rise to the conflict between constitutional rights and Pareto efficiency. With a minor reformulation of the theorem, it is possible, for instance, to show how predominantly negative externality inducing rights could render social choice contexts susceptible to the conflicts in question and how the condition stated in the theorem could resolve such conflicts. The theorem, however, which presents a sufficient (but not necessary) condition, is not meant to formulate, for all possible constitutional rights, an exhaustive set of conditions under which such conflicts could be eliminated. There may be cases where a society may reasonably grant to some individuals or groups (such as minorities) constitutional rights that may not satisfy the condition stated in the theorem. In such cases, some extra condition may need to be invoked to eliminate the conflicts in question.

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